# inkMaths

ANSWER BOOK

# The binomial distribution

Statistics · Year 12–13

8 questions · 22 marks

NAVY EDITION

## SECTION A · Warm-up

**A1** Every one of the twelve scores is equally likely, so X has a discrete uniform distribution on 1 to 12, with $P(X = x) = 1/12$ for each value. B1 for naming the discrete uniform distribution, B1 for 1/12 on every value. The specification asks you to recognise and name this distribution, not to compute with it — the word that earns the mark is **uniform**, and the reason is **equally** likely. A total of two dice is the standard trap: 7 is more likely than 2, so that variable is not uniform. [2]

**A2** A fixed number of trials n, each trial with exactly two outcomes, a constant probability of success p, and independence between trials. B1 for a fixed number of trials with two outcomes, B1 for a constant probability of success, B1 for independence. All four are needed, and independence is the one exam questions attack. [3]

**A3** $P(X = 2) = {}^{8}C_{2} \times 0.25^{2} \times 0.75^{6} = 28 \times 0.0625 \times 0.1780 = 0.311$. M1 for the complete product, A1 for 0.311. Three ingredients multiply in one line: the number of ways of placing the two successes, the probability of those successes, and the probability of the six failures. The exponents must sum to 8, which is the quickest check that the line is right. [2]

## SECTION B · Standard

**B1** $P(X = 3) = {}^{12}C_{3} \times 0.2^{3} \times 0.8^{9} = 220 \times 0.008 \times 0.1342 = 0.236$. $P(X = 0) = 0.8^{12} = 0.0687$, with no coefficient because there is only one way to fail every time. M1 for the product, A1 for 0.236, B1 for 0.0687. Sense check: $np = 2.4$, so a value of 3 should be common, and it is. The mode is 2, at 0.283, with 3 the next most likely outcome. [3]

**B2** $P(X \geq 4) = 1 - P(X \leq 3)$. B1 for the complement, B1 for the boundary dropping to 3. Calculators only sum upwards from zero, so every 'at least' converts to one minus an 'at most', with the boundary dropped by one. Writing $1 - P(X \leq 4)$ instead quietly removes the case $X = 4$ from the answer, and that off-by-one is the main hazard of the topic. [2]

**B3** $X \sim B(10, 0.1)$, and $P(X \leq 1) = P(X = 0) + P(X = 1) = 0.9^{10} + 10 \times 0.1 \times 0.9^{9} = 0.349 + 0.387 = 0.736$. M1 for adding the two point probabilities, A1 for 0.349 and 0.387, A1 for 0.736. With a boundary this low, summing the two point probabilities by hand shows the working more clearly than the calculator's cumulative function does, and either route earns the marks. [3]

**B4** Friends go to parties together, so one student answering yes makes their friends more likely to answer yes too. The trials are not independent, and independence is a binomial condition. B1 for naming independence, B1 for tying it to friends attending together. An answer has to name the condition and tie it to the context; 'the students are not independent' alone is too vague, so say why they are not. [2]

## SECTION C · Stretch

**C1** Let $X \sim B(n, 0.2)$. Then $P(X \geq 1) = 1 - P(X = 0) = 1 - 0.8^n$, so the requirement is $1 - 0.8^n > 0.99$, that is $0.8^n < 0.01$. Take logarithms of both sides: $n \log 0.8 < \log 0.01$. Since $\log 0.8$ is negative, the inequality reverses on dividing, giving $n > \log 0.01/\log 0.8 = 20.6$. So the smallest whole number is $n = 21$. Confirm it: $1 - 0.8^{20} = 0.9885$, which fails, and $1 - 0.8^{21} = 0.9908$, which passes. Marks go for turning 'at least one' into its complement, for forming the inequality, for reversing it correctly when dividing by a negative logarithm, and for rounding up rather than to the nearest whole number. In codes that is M1 for the complement, A1 for $0.8^n < 0.01$, M1 for taking logarithms, A1 for reversing the inequality, A1 for $n = 21$. Rounding 20.6 down to 20 is the loss the question is built around, and quoting the two probabilities is the cheapest way to prove you have not made it. [5]