



ANSWER BOOK

The binomial expansion

Sequences and series · Year 12–13

7 questions · 22 marks

NAVY EDITION

SECTION A · Warm-up

- A1** $8!/(3! \times 5!) = (8 \times 7 \times 6)/(3 \times 2 \times 1) = 336/6 = 56$. M1 for the cancelled quotient, [2]
A1 for 56. The 5! wipes out all but three factors of the top, so only $8 \times 7 \times 6$ survives.
Computing $8! = 40\,320$ in full is the long way round and invites arithmetic slips.
- A2** 1, 5, 10, 10, 5, 1, which is six entries for six terms. B1 for the row. Each entry is the [1]
sum of the two above it, and the row reads the same in both directions. A power of 5
needs the row starting 1, 5, so counting the apex as row 1 and reaching 1, 4, 6, 4, 1 is the
off-by-one to avoid.

SECTION B · Standard

- B1** Row 4 gives 1, 4, 6, 4, 1. Build each term with the powers of 3 falling and the [5]
powers of $2x$ climbing: 81 , then $4 \times 27 \times 2x = 216x$, then $6 \times 9 \times 4x^2 = 216x^2$, then $4 \times 3 \times 8x^3$
 $= 96x^3$, then $16x^4$. So $(3 + 2x)^4 = 81 + 216x + 216x^2 + 96x^3 + 16x^4$. Setting $x = 1$ turns the
bracket into $5^4 = 625$, and $81 + 216 + 216 + 96 + 16 = 625$, so the check passes for the
price of one line. B1 for the row of coefficients, M1 for a term with both powers correct, A1
A1 for the middle terms, A1 for the full expansion. Every term needs the 2 raised as well
as the x ; leaving $2x$ as 2 in the later terms is the usual collapse.
- B2** $1 + {}^8C_1(3x) + {}^8C_2(3x)^2 = 1 + 24x + 28 \times 9x^2 = 1 + 24x + 252x^2$. M1 for the binomial [3]
coefficients with $3x$ substituted whole, A1 for $24x$, A1 for $252x^2$. The replacement $3x$ goes
into the formula whole and gets squared whole. Forgetting to square the 3 gives $84x^2$
and costs the final mark, which is the single commonest loss in this topic.
- B3** The terms are $2^5 = 32$, then $5 \times 2^4x = 80x$, then $10 \times 2^3x^2 = 80x^2$. Setting $x = 0.002$: [5]
 $32 + 0.16 + 0.00032 = 32.16032$. The true value is $32.1603203\dots$, so the truncation is already
good to six decimal places; the discarded x^3 term is only 40×0.002^3 , three parts in ten
million. M1 for a term with the falling power of 2, A1 for $80x$, A1 for $80x^2$, M1 for substituting
 $x = 0.002$, A1 for 32.16032 .

SECTION C · Stretch

- C1** The x^2 term is ${}^6C_2 \times 2^4 \times (kx)^2 = 15 \times 16 \times k^2x^2 = 240k^2x^2$. Setting $240k^2 = 60$ gives [4]
 $k^2 = \frac{1}{4}$, so $k = \frac{1}{2}$ or $k = -\frac{1}{2}$. Both are wanted, since only k^2 appears and the sign cannot be
recovered from an even power. At $k = \frac{1}{2}$ the expansion opens $64 + 96x + 60x^2$, which
confirms the coefficient. M1 for the x^2 term of the expansion, A1 for $240k^2$, M1 for solving
 $240k^2 = 60$, A1 for both values of k . The mark at risk is the 2^4 : the power on the first term
drops as the power on the second climbs, and answers that leave it at 2^6 or omit it
entirely reach a different k .

- C2** The coefficient nC_r counts the ways of choosing which r brackets supply the second letter; choosing those r is the same act as choosing the other $n - r$ to supply the first. So ${}^nC_r = {}^nC_{n-r}$, and the row is a palindrome. B1 for choosing r brackets being the same act as choosing the other $n - r$, B1 for the resulting symmetry of the row.
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