inkMaths

ANSWER BOOK

The continuous uniform distribution

Further Statistics 2 · Year FM

6 questions · 26 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $f(x) = 1/(b - a)$ for $a \le x \le b$ and zero elsewhere. $F(x) = 0$ for $x < a$, $(x - a)/(b - a)$ for $a \le x \le b$, and 1 for $x > b$. B1 for the density, B1 B1 for the three branches of F. Both branches outside the interval are part of the answer. [3]

A2 Mean = $(5 + 17)/2$ = **11**. Variance = $(17 - 5)^2/12 = 144/12$ = **12**. Standard deviation = $\sqrt{12}$ = **3.46** to three significant figures. B1 for the mean, M1 for the variance formula, A1 for 12 and 3.46. That is about 29% of the range for any uniform distribution, since $1/\sqrt{12} = 0.289$. [3]

SECTION B · Standard

B1 The height is 1/12. $P(X < 8) = 3/12$ = **0.25**. $P(9 < X < 15) = 6/12$ = **0.5**. For the conditional probability, $P(X > 14 \text{ and } X > 11) = P(X > 14) = 3/12$, and $P(X > 11) = 6/12$, so the answer is $(3/12)/(6/12)$ = **0.5**. B1 for 0.25, B1 for 0.5, M1 for the conditional form, A1 for 0.5. [4]

B2 The error is equally likely anywhere in the rounding interval, so it follows **U(−5, 5)** with mean 0. Variance = $10^2/12 = 8.333$, so the standard deviation is **2.89**. An error above 4 in size covers two stretches of length 1 out of a range of 10, so the probability is 2/10 = **0.2**. B1 for U(−5, 5), M1 for the variance formula, A1 for 2.89, M1 for the two stretches of length 1, A1 for 0.2. [5]

B3 The wait follows U(0, 15), so the expected wait is the midpoint, **7.5** minutes. P(wait > 10) = 5/15 = **1/3**. B1 for U(0, 15), B1 for 7.5 minutes, B1 for 1/3. The model assumes arrival independent of the timetable, which fails for a passenger who has checked it. [3]

SECTION C · Stretch

C1 $F(x) = \int 1/(b - a)\,dt$ from a to x = $(x - a)/(b - a)$ for $a \le x \le b$. That integral only gives the middle branch, and a cumulative distribution function has to be defined for every real x, so state it in full: $F(x) = 0$ for $x < a$, $(x - a)/(b - a)$ for $a \le x \le b$, and 1 for $x > b$. The branches meet, since the middle one is 0 at a and 1 at b, so F is continuous and non-decreasing across the whole line. $E(X) = \int x/(b - a)\,dx$ from a to b = $(b^2 - a^2)/[2(b - a)]$. Factorising the difference of squares gives $(b + a)(b - a)/[2(b - a)]$ = **$(a + b)/2$**, the midpoint, which the symmetry of the rectangle would have predicted. [8]

For the variance, $E(X^2) = \int x^2/(b - a)\,dx$ from a to b = $(b^3 - a^3)/[3(b - a)]$. Factorising the difference of cubes gives $(b^2 + ab + a^2)/3$.

Then $\mathrm{Var}(X) = E(X^2) - [E(X)]^2 = (b^2 + ab + a^2)/3 - (a + b)^2/4$. Over a common denominator of 12 that is $[4b^2 + 4ab + 4a^2 - 3a^2 - 6ab - 3b^2]/12 = (a^2 - 2ab + b^2)/12$ = **$(b - a)^2/12$**.

M1 for integrating the density, A1 for F(x), M1 for $\int x/(b - a)\,dx$, A1 for the mean, M1 for $E(X^2)$, A1 for $(b^2 + ab + a^2)/3$, M1 for $E(X^2) - [E(X)]^2$, A1 for the printed variance. Both factorisations, of the difference of squares and of the difference of cubes, are where this derivation is usually abandoned. Write them out in full.