



QUESTION WORKBOOK

The quality of tests

Further Statistics 1 · Year FM

7 questions · 26 marks

NAVY SCREEN EDITION

SECTION A · Warm-up

A1 Define a Type I error and a Type II error. [2]

A2 For $X \sim B(10, p)$ with $H_0: p = 0.5$, $H_1: p < 0.5$ and critical region $X \leq 1$, find the size of the test. [3]

SECTION B · Standard

B1 A test of $H_0: p = 0.5$ against $H_1: p < 0.5$ uses $X \sim B(10, p)$ with critical region $X \leq 1$. Find the probability of a Type II error when $p = 0.2$, and the power at that value. [4]

- B2** A test of $H_0: p = 0.5$ against $H_1: p < 0.5$ uses $X \sim B(10, p)$ with critical region $X \leq 1$, and its power at $p = 0.2$ is 0.376. Find the power at $p = 0.1$ and comment on how the two compare. [3]

- B3** A Poisson test has $H_0: \lambda = 5$ against $H_1: \lambda > 5$ with critical region $X \geq 10$. Find its size, and its power when $\lambda = 8$. [4]

- B4** Explain why widening a critical region raises the power of a test but is not automatically an improvement. [3]

SECTION C · Stretch

- C1** $X \sim B(20, p)$ is used to test $H_0: p = 0.3$ against $H_1: p < 0.3$ at the 5% level. Given $P(X \leq 2 \mid p = 0.3) = 0.0355$ and $P(X \leq 3 \mid p = 0.3) = 0.1071$, find the critical region and the size of the test, then find the power at $p = 0.15$ and the probability of a Type II error there. [7]