inkMaths

ANSWER BOOK

The Simplex algorithm

Decision Mathematics 1 · Year FM

7 questions · 31 marks

NAVY EDITION

SECTION A · Warm-up

A1 Stop when no negative entries remain in the objective row. A negative entry means increasing that variable would still improve the objective, so while one remains the solution can be bettered. Once none remain, no variable can be increased without reducing the objective, and the current vertex is optimal. B1 for the stopping condition, B1 for the reason. [2]

A2 Add slack variables: $x + y + s_1 = 10$ and $2x + 3y + s_2 = 24$, with the objective as $P - 4x - 5y = 0$. The tableau rows are **P:** 1, −4, −5, 0, 0 | 0; **s_1:** 0, 1, 1, 1, 0 | 10; **s_2:** 0, 2, 3, 0, 1 | 24. B1 for the slack variables, B1 for the objective row, B1 B1 for the two constraint rows. [4]

SECTION B · Standard

B1 The most negative entry in the objective row is −5, so the pivot column is **y**. The ratio test gives 10 ÷ 1 = 10 and 24 ÷ 3 = 8, so the pivot row is the **s_2** row and the pivot element is 3. [6]
Divide that row by 3 to give the new y row **0,** 2/3, 1, 0, 1/3 | 8. Subtract it from the s_1 row for **0,** 1/3, 0, 1, −1/3 | 2, and add five times it to the objective row for **1,** −2/3, 0, 0, 5/3 | 40. So P = **40** with y = 8, x = 0 and s_1 = 2. B1 for the pivot column, M1 for the ratio test, A1 for the pivot row, M1 for the row operations, A1 for the new y row, A1 for the objective row. The objective row still has −2/3 in the x column, so a second iteration is needed. Name the row operations as R − R(y) and R + 5R(y) beside the tableau, since a mark goes on them.

B2 No entry in the objective row is negative, so no variable can be increased to improve P: the tableau is **optimal**. The basic variables are x = **6** and y = **4**, the non-basic slacks are $s_1 = s_2 = 0$, and P = **44**. B1 for the optimality statement, B1 for x = 6, B1 for y = 4, B1 for P = 44. Both slacks being zero means both constraints are tight, matching the graphical answer. [4]

B3 Ratios: 12 ÷ 2 = **6** and 20 ÷ 4 = **5**. The −1 is skipped entirely: increasing the entering variable would make that row's value grow rather than shrink, so it places no limit. The smallest ratio is 5, so the **third** row is the pivot row. M1 for the ratio test, A1 for the third row, B1 for ignoring the negative entry. [3]

SECTION C · Stretch

C1 The only negative entry left is -2/3, so the pivot column is **x**. The ratio test gives 2 ÷ 1/3 = **6** and 8 ÷ 2/3 = 12, so the pivot row is the **s_1** row and the pivot element is 1/3. Multiply that row by 3 to give the new x row **0,** 1, 0, 3, -1 **|** 6. Subtract two thirds of it from the y row for **0,** 0, 1, -2, 1 **|** 4, and add two thirds of it to the objective row for **1,** 0, 0, 2, 1 **|** 44. No negative entries remain, so the tableau is optimal: x = **6**, y = **4**, $s_1 = s_2 = 0$ and P = **44**. B1 for the pivot column, M1 for the ratio test, A1 for the pivot row, M1 for the row operations, A1 for the new x row, A1 for the other two rows, A1 for the solution. Negative entries away from the objective row are perfectly normal and are ignored by the ratio test. Only the pivot column entries matter there, and only the positive ones. [7]

C2 With only ≤ constraints and non-negative right-hand sides, the origin is feasible and the slack variables form a valid starting solution. A ≥ constraint with a positive right-hand side makes the origin infeasible, and subtracting a surplus variable does not replace what the slacks provided: setting the real variables to zero would need a negative surplus, so there is no obvious starting basis to read off. An artificial variable is added to each such row to supply one, and must then be driven out. The **two-stage** method first minimises the sum of the artificial variables, driving them to zero and so reaching a feasible corner, then runs ordinary Simplex from there. The **big-M** method instead subtracts a large multiple M of each artificial variable from the objective, making them so expensive that the algorithm eliminates them on its own. B1 B1 for why the ordinary start fails, B1 for the two-stage method, B1 for the big-M method, B1 for how the artificial variables are driven out. [5]