inkMaths

ANSWER BOOK

The stepping-stone method

Decision Mathematics 2 · Year FM

6 questions · 29 marks

NAVY EDITION

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SECTION A · Warm-up

A1 The index is the cell's cost minus its row shadow cost and its column shadow cost. It is the change in the total cost per unit sent along that route, once the rest of the plan has been adjusted to keep the supplies and demands satisfied. A positive index means using that route would raise the total, so the cell should stay empty. B1 for cost minus the two shadow costs, B1 for what the index measures, B1 for the positive case. [3]

A2 Index = 8 - 3 - 6 = **-1**. It is negative, so the route is cheaper than the shadow costs predict and the cell **is** a candidate. M1 for cost minus the shadow costs, A1 for -1, B1 for the conclusion. Whether it actually enters depends on whether any other unused cell has a more negative index. [3]

SECTION B · Standard

B1 Set R(A) = 0 and work outwards through the cells that are in use, since each of those satisfies R + K = cost. AX gives K(X) = 8 and AY gives K(Y) = 5. BY gives R(B) = 9 - 5 = **4**, CY gives R(C) = 6 - 5 = **1**, and CZ gives K(Z) = 5 - 1 = **4**. [6]

Each unused cell then has index cost - R - K. AZ = 6 - 0 - 4 = **2**. BX = 4 - 4 - 8 = **-8**. BZ = 7 - 4 - 4 = **-1**. CX = 3 - 1 - 8 = **-6**.

M1 for setting R(A) = 0 and working outwards, A1 A1 for the R and K values, M1 for cost - R - K, A1 A1 for the four indices.

Write the R values down the side of the table and the K values along the top. The shadow costs are worth marks in their own right, so they must appear even if only the indices are asked for.

B2 The most negative index is -8, so **BX** enters. The loop turns only at cells that are in use, alternating signs: BX plus, BY minus, AY plus, AX minus, back to BX. [6]

The minus cells hold BY 30 and AX 35, so the value moved is **30** and **BY** exits.

New allocations: AX 5, AY 35, BX 30, CY 10, CZ 40. Cost = 835 + 30(-8) = **595**, and directly 5(8) + 35(5) + 30(4) + 10(6) + 40(5) = 40 + 175 + 120 + 60 + 200 = 595.

B1 for BX entering, M1 for the stepping-stone loop, A1 for the alternating signs, A1 for moving 30, A1 for BY exiting, A1 for 595.

B3 The entering cell is chosen by improvement index, not by cost. The index already accounts for what the rest of the plan must give up to accommodate the new route, which the raw cost ignores. Here CX scored -6 against BX's -8, so a unit sent along BX saves more than one sent along CX even though CX looks cheaper on the table. B1 for the index rather than the cost deciding, B1 for what the index allows for, B1 for the comparison of -8 with -6. [3]

SECTION C · Stretch

C1 **CX** enters. The loop is CX plus, AX minus, AY plus, CY minus, back to CX. The minus cells hold AX 5 and CY 10, so the value moved is **5** and **AX** exits. [8]
New allocations: AY 40, BX 30, CX 5, CY 5, CZ 40. Cost = 595 + 5(−6) = **565**, and directly 200 + 120 + 15 + 30 + 200 = 565.
A third round of shadow costs gives R(A) 0, R(B) 2, R(C) 1, K(X) 2, K(Y) 5 and K(Z) 4, so the four remaining indices are AX 6, AZ 2, BY 2 and BZ 1. All are positive, so no unused route would reduce the total and **565** is optimal. B1 for CX entering, M1 for the loop, A1 for moving 5, A1 for AX exiting, A1 for the new allocations, A1 for 565, M1 for the third round of indices, B1 for the optimality statement. Stopping at 595 because the improvement looked small would have left 30 on the table.