inkMaths

ANSWER BOOK

The travelling salesman problem

Decision Mathematics 1 · Year FM

6 questions · 26 marks

NAVY EDITION

SECTION A · Warm-up

A1 The practical problem allows a node to be visited more than once; the classical problem visits each exactly once and needs a complete network obeying the triangle inequality. Replacing every pair's distance by the shortest path between them produces such a network, turning a practical problem into a classical one. B1 for the practical problem, B1 for the classical problem, B1 for the shortest-path conversion. [3]

A2 From A the nearest is B at 20. From B the nearest unvisited vertex is E at 28, then from E it is D at 22, then C at 12. The return from C to A costs 35, and it must be taken whatever its length. Tour **A** B E D C A of length **117**. This is an upper bound for the optimal tour, since it is a genuine tour. M1 for the nearest neighbour method, A1 for the tour, A1 for 117, B1 for the upper bound. [4]

SECTION B · Standard

B1 Remove A and everything joined to it, leaving B, C, D and E. Their minimum spanning tree takes CD 12, then DE 22, then BE 28, total **62**. The two shortest edges from A are AB 20 and AE 25, total **45**. Lower bound = 62 + 45 = **107**. [5]
M1 for deleting A, A1 for the tree edges, A1 for 62, B1 for the two shortest edges at A, A1 for 107.
No tour can be shorter. Deleting A from any tour leaves a path through the others, which is a spanning tree of that reduced network and so weighs at least 62, plus two edges at A which weigh at least 45. Name the tree edges as well as the total, since one mark is for the tree itself.

B2 20 + 30 + 12 + 22 + 25 = **109**, which is a better upper bound than the 117 nearest neighbour gave. So the optimal tour T satisfies **107** $\le T \le$ 109. Nearest neighbour was eight worse than this tour: it is quick and always gives a valid tour, but it is not reliable. M1 for summing the tour, A1 for 109, B1 for the bracket, B1 for the comment. [4]

B3 Each deletion gives a valid lower bound, but the bounds differ because the trees and the two shortest edges differ. Since every one is a genuine lower bound, the largest of them is the strongest statement available, so try several deletions and quote the largest result. B1 for every deletion giving a valid bound, B1 for why the bounds differ, B1 for quoting the largest. [3]

SECTION C · Stretch

C1 Remove C, leaving A, B, D and E with edges AB 20, AD 42, AE 25, BD 34, BE 28 and DE 22. The minimum spanning tree takes AB 20, then DE 22, then AE 25, total **67**. The two shortest edges from C are CD 12 and BC 30, total **42**. Lower bound = 67 + 42 = **109**. [7]

The tour A B C D E A has length 109, so 109 ≤ T ≤ 109 and the optimal tour is **109**, achieved by A B C D E A.

M1 for deleting C, A1 for the tree edges, A1 for 67, B1 for the two shortest edges at C, A1 for 109, M1 for comparing with the known tour, A1 for the optimal tour.

When a lower bound and a known tour agree, the tour is proved optimal and no further search is needed. Deleting A gave only 107, so the choice of vertex matters. Try each in turn and quote the largest bound, since every one of them is valid and the largest is the most informative.