InkMaths

ANSWER BOOK

Transportation problems

Decision Mathematics 2 · Year FM

6 questions · 24 marks

NAVY EDITION

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SECTION A · Warm-up

A1 Total supply is 120 and total demand is 105, so the problem is unbalanced. Add a **dummy** destination of demand **15**, reachable from every source at a cost of zero. B1 for the two totals, B1 for the dummy destination, B1 for a demand of 15. Whichever source ends up supplying the dummy is the one whose goods are not sent anywhere. [3]

A2 4 + 5 − 1 = **8** cells. Using fewer means the solution is **degenerate**, which happens when a supply and a demand run out together. M1 for 4 + 5 − 1, A1 for the cell count, B1 for degeneracy. A zero allocation is recorded in a suitable cell to restore the count, without moving any goods. [3]

SECTION B · Standard

B1 AX takes min(40, 35) = 35, using up X. AY then takes A's remaining 5, using up A. BY takes 30, using up B. CY takes the last 10 that Y needs, and CZ takes 40.
Allocations: **AX** 35, AY 5, BY 30, CY 10, CZ 40.
Cost = 35(8) + 5(5) + 30(9) + 10(6) + 40(5) = 280 + 25 + 270 + 60 + 200 = **835**. M1 for starting at the north-west corner, A1 A1 for the allocations, M1 for the cost sum, A1 for 835. [5]

B2 It uses five cells and 3 + 3 − 1 = 5, so the count is right and no zero allocation is needed. The cost is poor. The method never looks at a cost at all, and it has put 30 units on BY at 9 per unit, the dearest route in the table. B1 for the cell count, B1 for the comment on the cost. Its only purpose is to hand the improvement machinery a feasible plan to start from. [2]

B3 Give that cell a cost large enough that no improvement method would ever choose it, and say in your working that you have done so. A cost of zero would make the route look like the cheapest in the table, so the method would fill it first: the forbidden route needs to be made unattractive, not free. B1 for a prohibitively large cost, B1 for saying so in the working, B1 for why zero is wrong. A dummy destination is the opposite case, where zero is right because nothing is actually moved. [3]

SECTION C · Stretch

C1 Supply totals 90 and demand totals 75, so the problem is unbalanced. Add a **dummy** destination of demand 15, with a cost of zero from every source. [8]
North-west corner on the 3 by 4 table: AX takes min(30, 30) = **30**, which exhausts row A and column X together. Move on to BY, which takes min(40, 25) = **25**, then BZ takes the remaining **15** of B's supply, then CZ takes the **5** that Z still needs, and CD takes **15**.
Cost = 30(5) + 25(4) + 15(9) + 5(3) + 15(0) = 150 + 100 + 135 + 15 + 0 = **400**.
Only **five** cells are in use against 3 + 4 - 1 = **6**, so the solution is **degenerate**. It happened because row A and column X ran out at the same moment. Record a **zero** allocation in AY, next along the route, which restores the count without moving any goods and lets every shadow cost be found later.
B1 for the imbalance, B1 for the dummy demand of 15, M1 for the north-west corner method, A1 A1 for the allocations, M1 for the cost, A1 for 400, B1 for the degeneracy and the zero allocation.
Balance the problem before starting and count the cells after finishing. Both steps carry marks, and a degenerate table left uncorrected makes the shadow costs impossible.