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ANSWER BOOK

Trigonometric graphs and equations

Trigonometry · Year 12–13

7 questions · 25 marks

NAVY EDITION

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SECTION A · Warm-up

A1 sin 30° = ½, cos 45° = $\sqrt{2}/2$ and tan 60° = $\sqrt{3}$. B1 B1 B1 for the three exact values. [3]
All three come from two triangles, the halved equilateral and the halved square. On a non-calculator paper a decimal such as 0.866 is an approximation where the exact value $\sqrt{3}/2$ is asked for.

A2 The principal value is 60°, and cosine's symmetry supplies the partner 360° - 60° = 300°. So x = 60° or x = 300°. B1 for 60°, B1 for 300°. The cosine graph is symmetrical about x = 180° across this interval, so the two roots sit the same distance either side of that line. Quoting only the calculator value loses half the marks on every question of this shape. [2]

SECTION B · Standard

B1 The calculator returns -30°, which is outside the interval. Sine is negative in the third and fourth quadrants, so take the acute angle 30° and place it there: x = 180° + 30° = 210° and x = 360° - 30° = 330°. M1 for the acute angle 30°, A1 for 210°, A1 for 330°. Working from the size of the angle and then choosing quadrants is safer than trying to add 360° to the calculator value, which here gives 330° and hides the other root entirely. [3]

B2 Replace $\sin^2 x$ with $1 - \cos^2 x$, giving $2 - 2\cos^2 x + \cos x - 1 = 0$. That tidies to $2\cos^2 x - \cos x - 1 = 0$ and factorises as $(2\cos x + 1)(\cos x - 1) = 0$. From cos x = 1 comes x = 0°, and from cos x = -½ come x = 120° and x = 240°, so the solutions are 0°, 120° and 240°. M1 for using $\sin^2 x = 1 - \cos^2 x$, A1 for $2\cos^2 x - \cos x - 1 = 0$, M1 for factorising, A1 for x = 0°, A1 for 120° and 240°. The identity has to go in first; there is no way to factorise an equation carrying both sin and cos. Losing x = 0° because it sits on the boundary is the other frequent miss, and the interval does include it. [5]

B3 Gather everything on one side and factorise: sin x (cos x - 2) = 0. The bracket can never be zero, since cosine never reaches 2, so the only route left is sin x = 0, giving x = 0° or x = 180°. M1 for factorising, A1 for sin x (cos x - 2) = 0, A1 for 0°, A1 for 180°. Dividing both sides by sin x at the start destroys every solution the equation has, and it is the single most expensive move in this topic. Factorise, never cancel, when the thing you would cancel can be zero. [4]

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B4 Dividing by cos x is safe here, because $\cos x = 0$ would force $\sin x = \pm 1$ and leave the left side at ±3 against a right side of 0. So $\tan x = 2/3$, giving $x = 33.7°$ and, one half-turn along, 213.7°. M1 for $\tan x = 2/3$, A1 for 33.7°, A1 for 213.7°. Tangent repeats every 180°, so a tan equation delivers two roots in a 360° window and the second is found by adding 180°, not by subtracting from 360°. [3]

SECTION C · Stretch

C1 Let $\theta = x - 40°$, so θ runs over $-40° \le \theta < 320°$ while x covers its interval. $\cos\theta = \frac{1}{2}$ at $\theta = 60°$ and 300°, and also at −60°, which the widened interval excludes ($-60° < -40°$). Translating back: $x = 100°$ or 340°. M1 for the substitution $\theta = x - 40°$, B1 for the widened interval, A1 for $\theta = 60°$ and 300°, A1 for 100°, A1 for 340°. Widening the interval before hunting roots is the whole discipline; solving first and shifting after loses or invents solutions. [5]

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