InkMaths

ANSWER BOOK

Trigonometric modelling

Trigonometry · Year 12–13

7 questions · 24 marks

NAVY EDITION

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SECTION A · Warm-up

A1 The amplitude is 3 and the period is $2\pi/2 = \pi$ seconds. B1 for the amplitude, B1 for the period. The 2 inside the bracket speeds the wave up and lives in the period. The 3 outside scales the swing and lives in the amplitude. Quoting the period as 2 is the reflex to resist; the coefficient divides into 2π rather than being the answer. [2]

A2 The cosine swings between ±1, so h runs from $5 - 2 = 3$ up to $5 + 2 = 7$. B1 for the maximum, B1 for the minimum. The 5 sets the centre line and the 2 the swing about it. Both values are wanted, and the maximum on its own scores half. [2]

SECTION B · Standard

B1 The axle sits on the centre line, 10 m up, and the radius is the amplitude, 8 m. The seat tops out at $10 + 8 = 18$ m and bottoms at 2 m. The period is $2\pi \div (\pi/20) = 40$ s. B1 for the axle height, B1 for the radius, B1 for the maximum height, B1 for the period. The minus sign starts the seat at the bottom of the wheel at $t = 0$, where a ride starts. [4]

B2 Setting $10 - 8\cos(\pi t/20) = 14$ gives $\cos(\pi t/20) = -\frac{1}{2}$. The first positive angle with that cosine is $2\pi/3$, so $\pi t/20 = 2\pi/3$ and $t = 40/3 = 13.3$ s to 3 significant figures. M1 for $\cos(\pi t/20) = -\frac{1}{2}$, M1 for $\pi t/20 = 2\pi/3$, A1 for 13.3 s, B1 for taking the first crossing. Later crossings exist at $4\pi/3$ and beyond, but the question asks for the first. Take the principal value, then translate back through the coefficient, and keep the calculator in radians throughout. [4]

B3 The amplitude is 0.9 m and the period is $2\pi/0.6 = 10.5$ s. Mean sea level means $y = 0$. The buoy starts there at $t = 0$, and the sine next vanishes when $0.6t = \pi$, so $t = 5.24$ s, half a period later. B1 for the amplitude, B1 for the period, M1 for $0.6t = \pi$, A1 for 5.24 s. Reading the first positive time as 0 is the trap the word positive is there to close. [4]

SECTION C · Stretch

C1 $R = \sqrt{(4 + 2.25)} = 2.5$ and $\tan\alpha = 1.5/2$ gives $\alpha = 0.6435$, so $y = 2.5\sin(0.5t + 0.6435)$. High tide is 2.5 m, where the sine reaches 1: $0.5t + 0.6435 = \pi/2$, so $t = 1.85$ hours. M1 for the harmonic form, A1 for $R = 2.5$, M1 for $\tan\alpha$, A1 for $\alpha = 0.6435$, B1 for the high tide, A1 for $t = 1.85$ hours. Two terms, one tide: the harmonic form is the tool that makes the data readable. [6]

C2 Amplitude and period are separate dials. Doubling the amplitude doubles how far the oscillation swings and leaves its timing alone; oscillating faster needs a bigger coefficient inside the bracket, which shortens the period. B1 for amplitude and period being independent, B1 for what does change the speed. The claim muddles the outside dial with the inside one. [2]