



ANSWER BOOK

Vectors in two dimensions

Vectors · Year 12–13

7 questions · 22 marks

NAVY EDITION

SECTION A · Warm-up

- A1** $|a| = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$. The unit vector is a divided by its own length, $(5/13)i + (12/13)j$. B1 for 13, B1 for the unit vector. Its magnitude is 1 by construction, which makes a quick self-check. Dividing by 13 must reach both components, and halving that job is the usual slip. [2]
- A2** $2p + 3q = (4 - 3)i + (-6 + 12)j = i + 6j$. M1 for multiplying out both brackets, A1 for $i + 6j$. The arithmetic runs slot by slot, and the only traps are the signs. Multiply through each bracket before collecting. [2]
- A3** $|7i - 24j| = \sqrt{49 + 576} = \sqrt{625} = 25$. The angle satisfies $\tan \theta = 24/7$, so $\theta = 73.7^\circ$, measured below the positive x -direction because the j -component is negative. B1 for 25, M1 for $\tan \theta = 24/7$, A1 for the angle below the axis. Stating which side of the axis the vector lies is part of the answer, since a magnitude and a bare angle do not pin a vector down. [3]

SECTION B · Standard

- B1** $AB = b - a = (4 - (-2))i + (-5 - 3)j = 6i - 8j$, destination minus start. The distance is $|AB| = \sqrt{36 + 64} = \sqrt{100} = 10$. The midpoint averages the position vectors, giving $((-2 + 4)/2, (3 - 5)/2) = (1, -1)$. M1 for $b - a$, A1 for $6i - 8j$, A1 for the distance, B1 for the midpoint. Writing $a - b$ instead of $b - a$ reverses the vector and costs the first mark, though the distance survives. [4]
- B2** Parallel means one vector is a scalar multiple of the other, so $6i + \lambda j = k(4i + 6j)$. Comparing i -components gives $k = 6/4 = 3/2$, and then $\lambda = 6 \times 3/2 = 9$. Equivalently the components are in proportion, $4/6 = 6/\lambda$, which cross-multiplies to $4\lambda = 36$. M1 for setting up the scalar multiple, A1 for $k = 3/2$, A1 for $\lambda = 9$. Setting up the proportion upside down gives $\lambda = 4$, so check the answer against the pattern that both components grow by the same factor. [3]
- B3** $|5i - 12j| = \sqrt{25 + 144} = 13$, and $26 = 2 \times 13$, so scale by ± 2 . The vectors are $10i - 24j$ and $-10i + 24j$. M1 for the magnitude 13 and the scale factor 2, A1 for $10i - 24j$, A1 for $-10i + 24j$. Forgetting the negative one loses a mark, because parallel includes pointing the opposite way. A question asking for the same direction rather than parallel would want only the first. [3]

SECTION C · Stretch

C1 P is one third of the way from A to B, so $OP = a + \frac{1}{3}(b - a)$. Here $b - a = 15i + 6j$, [5]
so $\frac{1}{3}(b - a) = 5i + 2j$ and $OP = (1 + 5)i + (6 + 2)j = 6i + 8j$. Then $|OP| = \sqrt{36 + 64} = 10$. M1
for $OP = a + \frac{1}{3}(b - a)$, A1 for $\frac{1}{3}(b - a)$, A1 for $6i + 8j$, M1 for the magnitude, A1 for 10. Check
the ratio afterwards. $AP = 5i + 2j$ and $PB = 10i + 4j$, which is $2AP$, so $AP : PB = 1 : 2$ as
required. The ratio 1 : 2 splits AB into three equal parts, not two, and reading it as a half is
the error that dominates this question. The other trap is applying the fraction to b rather
than to $b - a$, which gives a point that does not lie on AB at all. The reliable form is $OP =$
 $a + [m/(m + n)](b - a)$ for a ratio $m : n$.
