inkMaths

ANSWER BOOK

Volumes of revolution

Further calculus · Year FM

7 questions · 26 marks

NAVY EDITION

SECTION A · Warm-up

A1 $V = \pi\int y^2\,dx$ from a to b. Each strip sweeps a disc of radius y and thickness dx, and a disc's volume is π times radius squared times thickness. B1 for the formula, B1 for the disc explanation. [2]

A2 $V = \pi\int 9\,dx$ from 0 to 4 = 36π. The solid is a cylinder of radius 3 and length 4, and $\pi r^2 h = \pi \times 9 \times 4 = 36\pi$ agrees. M1 for $\pi\int 9\,dx$, A1 for 36π, B1 for the cylinder check. Constant y is the quickest sanity check the formula ever gets. [3]

SECTION B · Standard

B1 $V = \pi\int x^4\,dx$ from 0 to 2 = $\pi[x^5/5] = 32\pi/5$. Square first, integrate second. M1 for squaring y, A1 for the integrand, M1 for integrating, A1 for 32π/5. The integrand is x^4, not x^2, and the power-rule slip of integrating y before squaring is the topic's classic error. [4]

B2 Radius is x with $x^3 = y$, so $x^2 = y^{2/3}$ and $V = \pi\int y^{2/3}\,dy$ from 0 to 8 = $\pi[3y^{5/3}/5] = \pi \times 3 \times 32/5 = 96\pi/5$. M1 for x^2 in terms of y, A1 for the integrand, M1 for integrating with the y-limits, A1 for 96π/5. About the y-axis everything transposes. The limits become y-limits, and x^2 has to be expressed in terms of y. [4]

B3 Outer radius x, inner radius x^2: $V = \pi\int(x^2 - x^4)\,dx$ from 0 to 1 = $\pi(1/3 - 1/5) = 2\pi/15$. M1 for the difference of squares, A1 for the integrand, M1 for integrating, A1 for 2π/15. Subtract the squares, never square the difference: $\pi\int(x - x^2)^2\,dx$ describes a different and wrong solid. [4]

B4 Each cross-section is an annulus, a disc of the outer radius with a disc of the inner radius removed, of area $\pi(\text{outer}^2 - \text{inner}^2)$. Squaring the difference instead computes a disc whose radius is the gap between the curves, which is a different region rotated about a different axis. B1 for the annulus, B1 for its area, B1 for what squaring the difference computes instead. [3]

SECTION C · Stretch

C1 The formula $V = \pi\int y^2\,dx$ still applies, with dx rewritten in terms of t. Here $dx/dt = 2t$, so $dx = 2t\,dt$, and $y^2 = 4t^2$. The limits transform as well, since x = 0 gives t = 0 and x = 4 gives t = 2. So $V = \pi\int 4t^2 \times 2t\,dt$ from 0 to 2 = $\pi\int 8t^3\,dt = \pi[2t^4]$ from 0 to 2 = **32π**. M1 for $dx = 2t\,dt$, B1 for the new limits, M1 for $y^2 = 4t^2$, A1 for $\pi\int 8t^3\,dt$, M1 for integrating, A1 for 32π. As a check, eliminating t gives $y^2 = 4x$, and $\pi\int 4x\,dx$ from 0 to 4 = $\pi[2x^2] = 32\pi$ as well. Leaving the limits in x while integrating with respect to t is the error that costs most marks on this type. [6]