



ANSWER BOOK

Diffraction gratings

Waves · Year 12

AQA 3.3.2.2 · CIE 8.4.1, 8.4.2 · OCR A 5.5.2(g), 5.5.2(h)

18 questions · 58 marks

NAVY EDITION

SECTION A · Warm-up

- A1** $d \sin \theta = n\lambda$ (1), where d is the grating spacing, the distance between adjacent slits, θ the angle of the maximum, n the order, and λ the wavelength (1). [2]
- A2** $d = 1/(300 \times 10^3 \text{ lines per metre})$ (1) [2]
 $d = 3.33 \times 10^{-6} \text{ m}$ (1)
- A3** A grating has many slits, so far more beams interfere (1). Constructive interference occurs only at precise angles where all beams are in phase, giving narrow, intense maxima; away from these angles the many beams cancel (1). [2]
- A4** The maximum in the straight-through direction, along the normal to the grating ($n = 0$) (1). The path difference between light from adjacent slits there is zero (1). [2]
- A5** The grating equation gives $\sin \theta = n\lambda/d$, and $\sin \theta$ cannot exceed 1 (1). Orders therefore exist only while $n\lambda/d \leq 1$, so no order beyond the whole-number part of d/λ has an angle at which it can form (1). [2]

SECTION B · Standard

- B1** $d = 1/(500 \times 10^3)$ (1) [4]
 $d = 2.00 \times 10^{-6} \text{ m}$ (1)
 $\sin \theta = n\lambda/d = (1 \times 600 \times 10^{-9})/(2.00 \times 10^{-6})$ (1)
 $\theta = 17.5^\circ$ (1)
- B2** $d = 1/(500 \times 10^3) = 2.00 \times 10^{-6} \text{ m}$ (1) [3]
 $\sin \theta = 2\lambda/d = (2 \times 600 \times 10^{-9})/(2.00 \times 10^{-6})$ (1)
 $\theta = 36.9^\circ$ (1)
- B3** $d = 1/(600 \times 10^3) = 1.67 \times 10^{-6} \text{ m}$ (1) [3]
 $\lambda = d \sin \theta = 1.67 \times 10^{-6} \times \sin 21^\circ$ (1)
 $\lambda = 597 \text{ nm}$ (1)
- B4** $d = \lambda/\sin \theta = (500 \times 10^{-9})/\sin 15^\circ$ (1) [3]
 $d = 1.93 \times 10^{-6} \text{ m}$ (1)
Lines per mm = $1/d \text{ per metre} \div 1000 = 518 \text{ lines per mm}$ (1)
- B5** $d = 1/(420 \times 10^3 \text{ lines per metre})$ (1) = $2.38 \times 10^{-6} \text{ m} \approx 2.4 \times 10^{-6} \text{ m}$ (1). $\sin \theta = 3\lambda/d$ [4]
 $= (3 \times 486 \times 10^{-9})/(2.38 \times 10^{-6}) = 0.612$ (1), so $\theta = 37.8^\circ$ (allow 37.4° to 37.8°) (1).

B6 $d = 1/(250 \times 10^3) = 4.00 \times 10^{-6} \text{ m}$ (1). $\sin\theta = \lambda/d = (633 \times 10^{-9})/(4.00 \times 10^{-6})$, so $\theta = 9.1^\circ$ (1). Distance of each spot from the centre $x = 1.5 \times \tan\theta = 0.24 \text{ m}$ (1). Separation of the two first-order spots $= 2x = 0.48 \text{ m}$ (1). [4]

B7 First-order angle from the centre $\theta = 40.0^\circ/2 = 20.0^\circ$ (1). $\lambda = d \sin\theta = 1.67 \times 10^{-6} \times \sin 20.0^\circ$ (1) $= 5.70 \times 10^{-7} \text{ m} = 570 \text{ nm}$ (1). [3]

SECTION C · Stretch

C1 The maximum order is where $\sin\theta = 1$, so $n_{\max} = d/\lambda$ (1) [4]
 $= (2.0 \times 10^{-6})/(600 \times 10^{-9}) = 3.33$ (1)
 The highest order is therefore $n = 3$ (1)
 At that order $\sin\theta = 3 \times 600 \times 10^{-9}/(2.0 \times 10^{-6})$, giving $\theta = 64.2^\circ$ (1)

C2 First order: $\sin\theta = \lambda/d$ (1) [4]
 For 400 nm, $\theta = 11.5^\circ$ (1)
 For 700 nm, $\theta = 20.5^\circ$ (1)
 Angular width $= 9.0^\circ$, with red diffracted more than violet (1)

C3 1. Measuring the wavelength of light or analysing spectra in a spectrometer (1). [2]
 2. Identifying elements from their line spectra, for example in the light from stars (1).

C4 $d = 1/(400 \times 10^3) = 2.5 \times 10^{-6} \text{ m}$ (1). Red end of the second order: $\sin\theta = (2 \times 700 \times 10^{-9})/d = 0.56$, so $\theta = 34.1^\circ$ (1). Violet end of the third order: $\sin\theta = (3 \times 400 \times 10^{-9})/d = 0.48$, so $\theta = 28.7^\circ$ (1). The third order begins at a smaller angle than the angle at which the second order ends, so the two spectra overlap (1). [4]

C5 Highest order = whole-number part of d/λ (1). Grating X: $d = 3.33 \times 10^{-6} \text{ m}$, $d/\lambda = 5.66$, so 5 orders are visible each side (1). Grating Y: $d = 1.25 \times 10^{-6} \text{ m}$, $d/\lambda = 2.12$, so only 2 orders are visible (1). Only X gives at least three orders, so the teacher should use grating X (1). [4]

C6 The central (zero-order) maximum is white, because at $n = 0$ the path difference is zero for every wavelength, so all colours arrive in phase there (1). Each non-zero order is drawn out into a continuous spectrum with violet nearest the centre and red furthest out (1), because $\sin\theta = n\lambda/d$, so the angle of a maximum increases with wavelength (1). $d = 1/(500 \times 10^3) = 2.0 \times 10^{-6} \text{ m}$, and the most demanding maximum is red in the second order: $\sin\theta = (2 \times 700 \times 10^{-9})/(2.0 \times 10^{-6}) = 0.70$, which is below 1, so the second-order spectrum runs the whole way to the red end (1). The first order spans 11.5° to 20.5° and the second 23.6° to 44.4° , so with this grating the two do not overlap (1). The second-order spectrum is the wider of the two and lies further from the centre, since doubling n doubles $\sin\theta$ for every wavelength (1). [6]