inkPhysics

ANSWER BOOK

Gravitational potential

Gravitational fields · Year 13

AQA 3.7.2.3 · CIE 13.4.1, 13.4.2, 13.4.3 · OCR A 5.4.4(a), 5.4.4(b), 5.4.4(c), 5.4.4(d)

18 questions · 46 marks

NAVY EDITION

SECTION A · Warm-up

A1 Work done per unit mass (1); in bringing a small test mass from infinity to the point (where the potential at infinity is zero) (1). [2]

A2 $V = -GM/r = -(6.67 \times 10^{-11} \times 6.0 \times 10^{24})/(6.4 \times 10^{6})$ (1)
$V = -6.25 \times 10^{7}\ \mathrm{J\ kg^{-1}}$ (1) [2]

A3 The potential is defined as zero at infinity and the gravitational force is attractive (1); work must be done against the field to move a mass to infinity, so at every point in the field the potential is below zero (1). [2]

A4 Zero (1); the potential does not change along an equipotential, so $\Delta W = m\Delta V = 0$ (1). [2]

A5 90° (they always cross at right angles) (1). [1]

SECTION B · Standard

B1 $V = -GM/r = -(6.67 \times 10^{-11} \times 6.0 \times 10^{24})/(1.0 \times 10^{7})$ (1)
$V = -4.00 \times 10^{7}\ \mathrm{J\ kg^{-1}}$ (1) [2]

B2 $V_{surface} = -GM/r = -6.25 \times 10^{7}\ \mathrm{J\ kg^{-1}}$ (1) [4]
$V_{final} = -3.13 \times 10^{7}\ \mathrm{J\ kg^{-1}}$ (1)
$\Delta V = 3.13 \times 10^{7}\ \mathrm{J\ kg^{-1}}$ (1)
$W = m\Delta V = 500 \times 3.13 \times 10^{7} = 1.56 \times 10^{10}\ \mathrm{J}$ (1)

B3 $\frac{1}{2}mv^2 = GMm/r$, so $v = \sqrt{(2GM/r)}$ (1) [3]
$v = \sqrt{(2 \times 6.67 \times 10^{-11} \times 6.0 \times 10^{24}/(6.4 \times 10^{6}))}$ (1)
$v = 1.12 \times 10^{4}\ \mathrm{m\ s^{-1}}$ (about $11.2\ \mathrm{km\ s^{-1}}$) (1)

B4 $g = \Delta V/\Delta r = (1.0 \times 10^{6})/(1.0 \times 10^{5})$ (1) [2]
$g = 10\ \mathrm{N\ kg^{-1}}$ (1)

B5 The area is the (magnitude of the) change in gravitational potential ΔV between the two distances (1) [3]
$W = m\Delta V = 800 \times 2.1 \times 10^{6}$ (1)
$W = 1.7 \times 10^{9}\ \mathrm{J}$ (1)

B6 The kinetic energy supplied must equal the energy needed to reach infinity: $\frac{1}{2}mv^2 = GMm/r$ (1); m cancels from both sides, giving $v = \sqrt{(2GM/r)}$, which is independent of the escaping mass (1). [2]

B7 ΔV is negative: the probe moves to a point of lower (more negative) potential (1). Gravitational potential energy is transferred to kinetic energy, the field doing work on the probe (1). [2]

SECTION C · Stretch

C1 To escape, the spacecraft must reach a point where V = 0 (1) [3]
$\Delta V = 0 - (-6.25 \times 10^7) = 6.25 \times 10^7$ J kg^{-1} (1)
$E = m\Delta V = 1000 \times 6.25 \times 10^7 = 6.25 \times 10^{10}$ J (1)

C2 $V_1 = -GM/r_1 = -5.72 \times 10^7$ J kg^{-1} (1) [3]
$V_2 = -2.86 \times 10^7$ J kg^{-1} (1)
$\Delta V = V_2 - V_1 = 2.86 \times 10^7$ J kg^{-1} (an increase) (1)

C3 The field strength equals the negative of the gradient of the potential-distance graph, $g = -\Delta V/\Delta r$ (1). The change in potential between two distances equals the area under the field strength-distance graph between them (1). Both g and the magnitude of V decrease with distance, g as $1/r^2$ and V as $1/r$ (1). [3]

C4 $W = m\Delta V$, with ΔV measured from the surface value (1) [4]
Orbit A: $W = 1500 \times 1.25 \times 10^7 = 1.88 \times 10^{10}$ J; orbit C: $W = 1.12 \times 10^{10}$ J, both within 2.0×10^{10} J (1)
Orbit B: $W = 1500 \times 1.75 \times 10^7 = 2.62 \times 10^{10}$ J, which exceeds the limit (1)
The vehicle can reach orbits A and C but not orbit B (1).

C5 $g = 6.0/4 = 1.5$ N kg^{-1} (1) [3]
$V = (-3.0 \times 10^7)/2 = -1.5 \times 10^7$ J kg^{-1} (1)
g is proportional to $1/r^2$ but V is proportional to $1/r$, so doubling r quarters g while only halving the magnitude of V (1).

C6 To just escape, $\frac{1}{2}mv^2 = m \times (0 - V)$, so $\frac{1}{2}v^2 = 2.8 \times 10^6$ J kg^{-1} (1) [3]
$v = \sqrt{(2 \times 2.8 \times 10^6)}$ (1)
$v = 2.37 \times 10^3$ m s^{-1} ≈ 2.4 km s^{-1} (1)