inkPhysics

ANSWER BOOK

Projectile motion

Mechanics · Year 12

AQA 3.4.1.4 · CIE 2.1.9 · OCR A 3.1.3(a), 3.1.3(b)

17 questions · 57 marks

NAVY EDITION

SECTION A · Warm-up

A1 Gravity acts only vertically, so it changes the vertical velocity but not the horizontal velocity (1). The horizontal motion is at constant velocity and the vertical motion is free fall; the two are independent and share only the time (1). [2]

A2 Fall time: $t = \sqrt{(2h/g)} = \sqrt{(2 \times 1.2/9.81)}$ (1) [3]
$t = 0.495$ s (1)
Horizontal distance: $x = 3.0 \times 0.495 = 1.48$ m (1)

A3 The horizontal component stays constant (1); the vertical component changes uniformly at g, decreasing on the way up and increasing on the way down (1). [2]

SECTION B · Standard

B1 (a) $t = \sqrt{(2 \times 45/9.81)}$ (1) [4]
$t = 3.03$ s (1)
(b) $x = 20 \times 3.03 = 60.6$ m (1)
(c) $v_y = gt = 9.81 \times 3.03 = 29.7\ \mathrm{m\ s^{-1}}$ (1)

B2 (a) $u_x = 25 \cos 30° = 21.7\ \mathrm{m\ s^{-1}}$ (1) [5]
$u_y = 25 \sin 30° = 12.5\ \mathrm{m\ s^{-1}}$ (1)
(b) $T = 2u_y/g$ (1)
$T = 2.55$ s (1)
(c) $R = u_xT = 55.2$ m (1)

B3 $H = u_y^2/2g$ (1) [3]
$H = 12.5^2/(2 \times 9.81)$ (1)
$H = 7.96$ m (1)

SECTION C · Stretch

C1 $2\theta = 80°$ (1) [4]
$R = 30^2 \times \sin 80° / 9.81$ (1)
$R = 90.3$ m (1)
The maximum range for a given speed occurs at 45° (1)

C2 $u_x = 13.59$ m s^{-1} (1) [5]
$u_y = 6.34$ m s^{-1} (1)
Vertical: $0 = 2.0 + u_y t - ½gt^2$ (1)
$t = 1.55$ s (1)
Range $= u_x t = 21.1$ m (1)

C3 Air resistance opposes the motion, reducing both horizontal and vertical speeds (1). The path becomes asymmetric, steeper on the way down (1). The maximum height and range are reduced, and the angle for maximum range falls below 45° (1). [3]

SECTION A · Warm-up

A4 (a) Zero (1). (b) $g = 9.81$ m s^{-2}, directed vertically downwards; gravity acts throughout the flight (1). [2]

A5 They land at the same time (1). The vertical motions are identical, both falling from rest at g; the horizontal velocity has no effect on the vertical motion (1). [2]

SECTION B · Standard

B4 Fall time from the vertical motion: $t = \sqrt{(2 \times 3.2/9.81)} = 0.81$ s (1). Horizontally the speed is constant: $u = x/t = 7.2/0.808$ (1) $= 8.9$ m s^{-1} (1). [3]

B5 Vertical component at entry: $v_y = \sqrt{(2gh)} = \sqrt{(2 \times 9.81 \times 30)} = 24.3$ m s^{-1} (1). The horizontal component is unchanged at 12 m s^{-1}, so $v = \sqrt{(12^2 + 24.3^2)}$ (1) $= 27$ m s^{-1} (1). tan $\theta = 24.3/12$, so $\theta = 64°$ below the horizontal (1). [4]

B6 Horizontal: unchanged at 24 cos 55° = 13.8 m s^{-1} (1). Vertical: $v_y = 24 \sin 55° - 9.81 \times 1.5 = 4.9$ m s^{-1} (1). v_y is still positive (upwards), so the ball is still rising (1). [3]

SECTION C · Stretch

C4 At the highest point the vertical component of velocity is zero, but the horizontal component is unchanged throughout the flight (1). Speed = 26 cos 40° (1) = 19.9 m s^{-1} (1). [3]

C5 Components: $u_x = 22 \cos 38° = 17.3$ m s^{-1}, $u_y = 22 \sin 38° = 13.5$ m s^{-1} (1). Time to reach the posts: $t = 43/17.34 = 2.480$ s (1). Height then: $y = u_y t - ½gt^2 = 13.54 \times 2.480 - 4.905 \times 2.480^2$ (1) $= 3.4$ m (1). 3.4 m is above the 3.0 m crossbar, so the kick scores, clearing it by about 0.4 m (1). [5]

C6 Fall time: $t = \sqrt{(2 \times 300/9.81)} = 7.8$ s (1). Release distance = 65×7.8 (1) = 508 m ≈ 510 m before the target (1). The aircraft is directly above the package when it lands: both keep the same constant horizontal velocity of 65 m s^{-1} (1). [4]