



ANSWER BOOK

Rotational motion and moment of inertia

Engineering physics · Year 13

AQA 3.11.1.1, 3.11.1.2, 3.11.1.3

17 questions · 51 marks

NAVY EDITION

SECTION A · Warm-up

- A1** Angular velocity is the angle swept out per unit time about the axis, $\omega = \Delta\theta/\Delta t$ [2]
(1). Its unit is rad s^{-1} (1). Every point of a rigid body shares the same ω , however far from the axis it sits.
- A2** One revolution is 2π rad and one minute is 60 s, so $\omega = 2400 \times 2\pi/60$ (1) = 251 [2]
 rad s^{-1} (1). Leaving the speed in rev min^{-1} is the standard error here, and it poisons every line that follows.
- A3** It measures the body's resistance to angular acceleration about that axis, the [2]
rotational counterpart of mass (1). It depends on how much mass the body has and on how that mass is distributed about the axis, since $I = \sum mr^2$ weights each particle by the square of its distance (1).
- A4** $I = \sum mr^2$, so each particle counts by the square of its distance from the axis (1). [2]
The hoop carries all its mass at the full radius, while the disc spreads its mass inward where r^2 is small, so the hoop's sum is larger, in fact twice as large (1).
- A5** Displacement s becomes angular displacement θ ; velocity v becomes [2]
angular velocity ω ; acceleration a becomes angular acceleration α (1); mass m becomes moment of inertia I (1).
- A6** $E_k = \frac{1}{2}I\omega^2$ (1), the exact twin of $\frac{1}{2}mv^2$ with I standing in for m and ω for v . [1]

SECTION B · Standard

- B1** Convert first: $\omega_2 = 900 \times 2\pi/60 = 94.2 \text{ rad s}^{-1}$ (1). $\alpha = (\omega_2 - \omega_1)/t$ (1) = $94.2/6.0 =$ [3]
 15.7 rad s^{-2} (1). Dividing 900 by 6.0 without converting gives 150, a number in the wrong units entirely.
- B2** $\omega = 45 \times 2\pi/60 = 4.71 \text{ rad s}^{-1}$ (1). $E_k = \frac{1}{2}I\omega^2$ (1) = $\frac{1}{2} \times 0.045 \times 4.71^2 = 0.50 \text{ J}$ (1). The [3]
conversion is again the marked step; a raw 45 in the formula inflates the energy about ninety-fold.
- B3** Each mass sits at $r = 0.80/2 = 0.40 \text{ m}$, not 0.80 m (1). $I = \sum mr^2 = 2 \times 0.60 \times 0.40^2$ [3]
= 0.192 kg m^2 (1). At 0.10 m, $I = 2 \times 0.60 \times 0.10^2 = 0.012 \text{ kg m}^2$ (1), smaller by a factor of 16, because r^2 rewards distance so heavily. Using the rod's full length as r is the standard slip.

- B4** $\theta = 250 \times 2\pi = 1571 \text{ rad (1)}$. Use $\omega_2^2 = \omega_1^2 + 2\alpha\theta$ (1): $0 = 120^2 + 2\alpha \times 1571$, so $\alpha = -4.58 \text{ rad s}^{-2}$, a deceleration of 4.58 rad s^{-2} (1). Revolutions must become radians before they enter an angular SUVAT equation. [3]
- B5** Halve the diameter: $r = 0.90/2 = 0.45 \text{ m (1)}$. $v = \omega r = 12 \times 0.45 = 5.4 \text{ m s}^{-1}$ (1). $a = \alpha r = 3.0 \times 0.45 = 1.35 \text{ m s}^{-2}$ (1). Feeding the diameter straight in doubles both answers, and it is this option's most reliable lost mark. [3]
- B6** $I = \frac{1}{2} \times 25 \times 0.30^2 = 1.125 \text{ kg m}^2$ (1). $\omega = 2000 \times 2\pi/60 = 209 \text{ rad s}^{-1}$ (1). $E_k = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 1.125 \times 209^2 = 2.5 \times 10^4 \text{ J (1)}$. Both conversions, rev min^{-1} to rad s^{-1} and mass to I , must happen before the energy formula. [3]
- B7** $\theta = \omega_1 t + \alpha t^2/2$ (1) $= 0 + 4.0 \times 8.0^2/2 = 128 \text{ rad (1)}$. Revolutions $= 128/2\pi = 20.4$ (1). Dividing by 2π is the return trip of the usual conversion, and forgetting it leaves an answer 6.3 times too large. [3]

SECTION C · Stretch

- C1** The store is $E_k = \frac{1}{2}I\omega^2$. Doubling ω quadruples the energy, because ω is squared (1), while doubling the mass only doubles I and so only doubles the energy (1). Placing mass at the rim raises I for the same mass, because $I = \sum mr^2$ counts each particle by the square of its distance from the axis (1), so a rim-loaded wheel stores more energy at the same spin rate than a solid one of equal mass (1). Real flywheel batteries are therefore modest rim-heavy discs spun very fast in a vacuum, not heavy ones spun slowly. [4]
- C2** $I = \frac{1}{2} \times 180 \times 0.50^2 = 22.5 \text{ kg m}^2$ (1). $\omega_1 = 3600 \times 2\pi/60 = 377 \text{ rad s}^{-1}$ and $\omega_2 = 1800 \times 2\pi/60 = 188 \text{ rad s}^{-1}$ (1). $E_1 = \frac{1}{2} \times 22.5 \times 377^2 = 1.60 \times 10^6 \text{ J (1)}$. $E_2 = \frac{1}{2} \times 22.5 \times 188^2 = 4.00 \times 10^5 \text{ J (1)}$. Usable energy $= 1.60 \times 10^6 - 4.00 \times 10^5 = 1.20 \times 10^6 \text{ J (1)}$. Time $= 1.20 \times 10^6/40\,000 = 30 \text{ s (1)}$. Note that halving the spin rate empties three quarters of the store, not half, because $E_k \propto \omega^2$: an answer of half the energy has forgotten the square. Both speeds must also be converted from rev min^{-1} first. [6]
- C3** Disc: $I = \frac{1}{2} \times 60 \times 0.40^2 = 4.8 \text{ kg m}^2$, so $E_k = \frac{1}{2} \times 4.8 \times 150^2 = 5.4 \times 10^4 \text{ J (1)}$. Ring: $I = 60 \times 0.40^2 = 9.6 \text{ kg m}^2$, so $E_k = \frac{1}{2} \times 9.6 \times 150^2 = 1.1 \times 10^5 \text{ J (1)}$. The ring stores twice as much for the same mass and the same spin rate (1), so the ring is the better store: mass held at full radius counts fully in $\sum mr^2$, whereas the disc's inner mass contributes little (1). [4]
- C4** $\omega_2^2 = \omega_1^2 + 2\alpha\theta$ gives $\alpha = (20^2 - 8.0^2)/(2 \times 42)$ (1) $= 4.0 \text{ rad s}^{-2}$ (1). Then $t = (\omega_2 - \omega_1)/\alpha = 12/4.0 = 3.0 \text{ s (1)}$. Energy: $\Delta E_k = \frac{1}{2}I(\omega_2^2 - \omega_1^2)$ (1), so $1680 = \frac{1}{2} \times I \times (400 - 64)$ and $I = 10.0 \text{ kg m}^2$ (1). The energy step must use the difference of the squares; using $\frac{1}{2}I(\Delta\omega)^2$ with $\Delta\omega = 12$ is the standard error and gives an I more than twice too large. [5]