InkPhysics

ANSWER BOOK

Simple harmonic motion

Periodic motion · Year 13

AQA 3.6.1.2 · CIE 17.1.1, 17.1.2, 17.1.3, 17.1.4, 17.1.5
OCR A 5.3.1(a), 5.3.1(b), 5.3.1(c)(i), 5.3.1(d), 5.3.1(e), 5.3.1(f), 5.3.1(g)

19 questions · 55 marks

NAVY EDITION

SECTION A · Warm-up

A1 The acceleration is directly proportional to the displacement from the equilibrium position (1) and is always directed towards it: $a = -\omega^2 x$ ($a \propto -x$) (1). [2]

A2 $\omega = 2\pi f = 2\pi \times 2.0$ (1)
$\omega = 12.57\ \text{rad s}^{-1}$ (1) [2]

A3 $v_{max} = \omega A = 10 \times 0.050$ (1)
$v_{max} = 0.50\ \text{m s}^{-1}$ (1) [2]

A4 $a = \omega^2 x = 7.0^2 \times 0.030 = 1.5\ \text{m s}^{-2}$ (1). It is directed towards the equilibrium position, opposite to the displacement (1). [2]

A5 (a) At maximum displacement, $x = \pm A$, the extremes of the motion (1). (b) At zero displacement, the equilibrium position (1). [2]

A6 $f = 50/40 = 1.25$ Hz (1)
$T = 1/f = 0.80$ s (1) [2]

SECTION B · Standard

B1 $\omega = 2\pi f = 2\pi \times 3.0$ (1)
$\omega = 18.85\ \text{rad s}^{-1}$ (1)
$a_{max} = \omega^2 A = 7.11\ \text{m s}^{-2}$ (1) [3]

B2 $v = \pm\omega\sqrt{(A^2 - x^2)}$ (1)
$= 8.0 \times \sqrt{(0.10^2 - 0.060^2)}$ (1)
$v = 0.64\ \text{m s}^{-1}$ (1) [3]

B3 $x = A \cos \omega t = 0.15 \times \cos(4.0 \times 0.20)$ (1)
$= 0.15 \times \cos(0.80)$, with the calculator in radians (1)
$x = 0.105$ m (1) [3]

B4 $f = \omega/2\pi = 6.28/(2\pi) = 1.00$ Hz (1)
$T = 1/f = 1.00$ s (1) [2]

B5 $\omega = 2\pi f = 2\pi \times 0.50 = 3.14\ \text{rad s}^{-1}$ (1)
$x = 0.12 \times \sin(3.14 \times 0.40)$, with the calculator in radians (1)
$x = 0.114$ m (1) [3]

B6 $\omega = 2\pi f = 2\pi \times 250 = 1571$ rad s^{-1} (1) [4]
$a_{max} = \omega^2 A = 1571^2 \times 3.5 \times 10^{-4}$ (1)
$a_{max} = 864$ m s^{-2} (1)
50g = 490 m s^{-2}, and 864 m s^{-2} is larger, so the claim is correct (1)

B7 $\cos \omega t = 0.020/0.080 = 0.25$ (1) [3]
$\omega t = \cos^{-1}(0.25) = 1.32$ rad, with $\omega = 2\pi/1.2 = 5.24$ rad s^{-1} (1)
$t = 1.32/5.24 = 0.25$ s (1)

SECTION C · Stretch

C1 (a) $\omega = 2\pi/T = 2\pi/0.50$ (1) [4]
$\omega = 12.57$ rad s^{-1} (1)
(b) $v_{max} = \omega A = 1.01$ m s^{-1} (1)
(c) $a_{max} = \omega^2 A = 12.6$ m s^{-2} (1)

C2 $v = ½v_{max}$: $½\omega A = \omega\sqrt{(A^2 - x^2)}$ (1) [4]
$¼A^2 = A^2 - x^2$ (1)
$x = (\sqrt{3}/2)A$ (1)
$x = 0.087$ m (1)

C3 Velocity is a quarter of a cycle (90°) ahead of displacement (1), and is greatest when displacement is zero (1). Acceleration is exactly out of phase (180°) with displacement, being greatest and opposite when displacement is at a maximum (1). [3]

C4 a/x is the same for every pair: $-0.50/0.020 = -1.00/0.040 = -1.50/0.060 = -25$ s^{-2}, so $a \propto x$ (1) [4]
The acceleration is opposite in sign to the displacement, that is directed towards equilibrium, so the motion is SHM (1)
Comparing with $a = -\omega^2 x$ gives $\omega^2 = 25$ s^{-2} (1)
$\omega = 5.0$ rad s^{-1} (1)

C5 $A/2 = A \sin \omega t$ gives $\sin \omega t = ½$ (1) [3]
The first solution is $\omega t = \pi/6$ (1)
With $\omega = 2\pi/T$, $t = (\pi/6)/(2\pi/T) = T/12$ (1)

C6 $a_{max}/v_{max} = \omega^2 A/(\omega A) = \omega$ (1) [4]
$\omega = 14/0.90 = 15.6$ rad s^{-1} (1)
$A = v_{max}/\omega = 0.90/15.6 = 0.058$ m (1)
$f = \omega/2\pi = 2.5$ Hz (1)