inkPhysics

ANSWER BOOK

The time constant and exponential decay

Capacitance · Year 13

AQA 3.7.4.4 · CIE 19.3.1, 19.3.2, 19.3.3 · OCR A 6.1.3(b), 6.1.3(c), 6.1.3(d), 6.1.3(e)

18 questions · 49 marks

NAVY EDITION

SECTION A · Warm-up

A1 $\tau = RC$ (1); the time for the charge (or pd, or current) to fall to $1/e$ (about 37%) of its initial value (1). [2]

A2 $\tau = RC = 10000 \times 100 \times 10^{-6}$ (1)
$\tau = 1.0$ s (1) [2]

A3 $Q = Q_0 e^{-t/RC}$, where Q_0 is the initial charge (1). [1]

A4 63% (the fraction $1 - 1/e$) (1). [1]

A5 $\tau = RC = 2.2 \times 10^6 \times 47 \times 10^{-6}$ (1)
$\tau = 103$ s (1) [2]

SECTION B · Standard

B1 $\tau = RC = 2200 \times 470 \times 10^{-6}$ (1)
$\tau = 1.03$ s (1)
After one time constant, $Q/Q_0 = 1/e \approx 0.37$ (37%) (1). [3]

B2 $\tau = RC = 47000 \times 100 \times 10^{-6} = 4.7$ s (1)
$V = V_0 e^{-t/RC} = 12 \times e^{-5.0/4.7}$ (1)
$V = 4.14$ V (1) [3]

B3 $T_{½} = 0.69RC = 0.69 \times 4.7$ (1)
$T_{½} = 3.24$ s (1) [2]

B4 $R = \tau/C = 2.0/(220 \times 10^{-6})$ (1)
$R = 9091\ \Omega$ (9.1 kΩ) (1) [2]

B5 $V = V_0 e^{-t/\tau}$, so $t/\tau = \ln(V_0/V)$ (1)
$\ln(6.0/2.2) = 1.00$ (1)
$\tau = 15/1.00 = 15.0$ s ≈ 15 s (1) [3]

B6 $T_{½} = 0.69RC$ (1)
$C = T_{½}/(0.69R) = 4.6/(0.69 \times 33000)$ (1)
$C = 2.0 \times 10^{-4}$ F (202 μF) (1) [3]

SECTION C · Stretch

C1 $5.0 = 20e^{-t/3.0}$, so $e^{-t/3.0} = 0.25$ (1) [3]
$t = -3.0 \times \ln(0.25)$ (1)
$t = 4.16$ s (1)

C2 $\tau = RC = 10000 \times 470 \times 10^{-6} = 4.7$ s (1) [3]
$V = V_0(1 - e^{-t/RC}) = 9.0 \times (1 - e^{-10/4.7})$ (1)
$V = 7.93$ V (1)

C3 Taking logs of $V = V_0e^{-t/RC}$ gives $\ln V = \ln V_0 - t/RC$ (1); this is a straight line with gradient $-1/RC$ (1); so the time constant RC is the negative reciprocal of the gradient (1). [3]

C4 The gradient is $-1/RC$, so $RC = 1/0.185 = 5.4$ s (1) [4]
$C = RC/R = 5.4/10000 = 5.41 \times 10^{-4}$ F = 541 μF (1)
The tolerance allows 376 μF to 564 μF (1)
541 μF lies inside this range, so the capacitor is within its stated tolerance (1).

C5 $E = \frac{1}{2}Q^2/C$, so E is proportional to Q^2 (1) [3]
After one time constant Q has fallen to Q_0/e, so E falls to E_0/e^2 (1)
$1/e^2 = 0.135$, which is about 14% (1)

C6 $Q/Q_0 = 0.010 = e^{-t/\tau}$ (1) [3]
$t = -4.0 \times \ln(0.010) = 4.0 \times \ln(100)$ (1)
$t = 18.4$ s (about 4.6 time constants) (1)

C7 Charge the capacitor from a supply, then discharge it through the known resistor with a voltmeter or datalogger connected across the capacitor (or resistor) (1). Record the pd at regular time intervals, using a stopwatch (or the datalogger's clock) started as the discharge begins (1). Plot ln V against t (1); a straight line confirms that the decay is exponential (1). The gradient of the line equals $-1/RC$ (1); so $C = -1/(R \times \text{gradient})$, using the known resistance (1). [6]