



ANSWER BOOK

Uncertainty and error

Measurements · Year 12

AQA 3.1.2 · CIE 1.3.1, 1.3.2, 1.3.3

OCR A 1.1.3(c), 1.1.4(d), 1.1.4(e), 2.2.1(a), 2.2.1(b), 2.2.1(c), 2.2.1(d)

19 questions · 60 marks

NAVY EDITION

SECTION A · Warm-up

- A1** Random errors scatter readings unpredictably above and below the true value, and are reduced by repeating and averaging (1). Systematic errors shift all readings the same way, a zero error for example, and are not reduced by repeating (1). [2]
- A2** Percentage uncertainty = $(0.05/2.50) \times 100$ (1)
= 2.0% (1) [2]
- A3** Precision is how close repeated readings are to each other, that is how small the scatter is (1). Accuracy is how close readings are to the true value (1). Measurements can be precise but inaccurate if a systematic error is present. [2]
- A4** Resolution is the smallest change in the quantity that the instrument can display (1). For this stopwatch the resolution is 0.01 s (1). [2]
- A5** The first student's results are repeatable: the same experimenter with the same equipment gets consistent results (1). The agreement between the two students, using different equipment, shows the results are reproducible (1). [2]

SECTION B · Standard

- B1** When adding, absolute uncertainties add (1)
Total = 8.0 ± 0.3 m (1) [2]
- B2** Percentage uncertainties: length 2%, width 2% (1)
For a product they add, so the area uncertainty is 4% (1)
Area = $10.0 \times 5.0 = 50$ cm² (1)
Uncertainty = 4% of 50 = ± 2 cm² (1) [4]
- B3** Uncertainty is ± 0.5 mm on a reading of 45 mm (1)
Percentage uncertainty = $(0.5/45) \times 100$ (1)
= 1.1% (1) [3]
- B4** Mean = $(20.1 + 20.3 + 20.2 + 20.4)/4$ (1)
Mean = 20.25 mm (1)
Uncertainty $\approx \text{range}/2 = (20.4 - 20.1)/2 = \pm 0.15$ mm (1) [3]
- B5** (a) A systematic (zero) error (1). (b) Corrected diameter = $0.36 - (-0.03) = 0.39$ mm (1). (c) The offset shifts every reading the same way, so averaging repeats leaves the shift unchanged; the correction must be applied instead (1). [3]

B6 (a) When subtracting, absolute uncertainties add: extension = 6 ± 2 mm (1). [4]
 (b) Evidence of use of (uncertainty/value) $\times 100$, i.e. $(2/6) \times 100$ (1); percentage uncertainty = 33% (1). (c) Use a larger load so the extension is bigger (or measure the extension directly between fixed marks): the same ± 2 mm is then a smaller fraction of the value (1).

B7 Percentage uncertainty in $l = (0.002/0.650) \times 100 = 0.3\%$ (1). T is squared, so its [3]
 percentage uncertainty counts twice: $2 \times (0.02/1.62) \times 100 = 2.5\%$ (1). Total = $0.3 + 2.5 = 2.8\%$ (1).

SECTION C · Stretch

C1 Percentage uncertainty in $r = (0.1/4.0) \times 100 = 2.5\%$ (1) [4]
 Area $\propto r^2$, so its percentage uncertainty = $2 \times 2.5\% = 5\%$ (1)
 Area = $\pi \times 4.0^2 = 50.3 \text{ cm}^2$ (1)
 Uncertainty = $\pm 2.5 \text{ cm}^2$ (1)

C2 Percentage uncertainties: mass 1%, volume 5% (1) [4]
 For a quotient they add, giving 6% (1)
 $\rho = 0.500/(2.0 \times 10^{-4}) = 2500 \text{ kg m}^{-3}$ (1)
 Uncertainty = $6\% = \pm 150 \text{ kg m}^{-3}$ (1)

C3 (a) Reduce random error by repeating readings and taking a mean (1), and by [3]
 using instruments of finer resolution (1). (b) Reduce systematic error by checking for
 and correcting zero errors, calibrating the instrument, and improving the technique (1).

C4 Uncertainty = half the difference between the extreme gradients (1): $(0.212 -$ [4]
 $0.196)/2 = \pm 0.008 \text{ s}^2 \text{ m}^{-1}$ (1). Percentage uncertainty = $(0.008/0.204) \times 100 = 3.9\%$ (1).
 Gradient = $0.204 \pm 0.008 \text{ s}^2 \text{ m}^{-1}$ (1).

C5 Mean = $(5.11 + 5.09 + 5.07 + 5.09)/4 = 5.09 \text{ mm}$ (1), with uncertainty = half the [4]
 range = $\pm 0.02 \text{ mm}$ (1). The claim allows $5.00 \pm 0.05 \text{ mm}$, i.e. 4.95 to 5.05 mm (1). Even
 the smallest reading (5.07 mm) lies above 5.05 mm, so the measured range is wholly
 outside the tolerance and the bearing does not meet the claim (1).

C6 h contributes $(0.005/1.250) \times 100 = 0.4\%$ (1). t is squared, so it contributes $2 \times$ [3]
 $(0.005/0.505) \times 100 = 2.0\%$ (1). The timing contributes about five times the percentage
 uncertainty of the height, so the measurement of t most needs improving (1).

- C7** Draw the best-fit straight line and measure its gradient using a large triangle [6]
(1). Comparing with $T^2 = (4\pi^2/g)l$, the gradient equals $4\pi^2/g$ (1), so $g = 4\pi^2/\text{gradient}$ (1).
Draw the steepest and shallowest lines that still pass through all the error bars and
measure their gradients (1). The uncertainty in the gradient is half the difference
between these two gradients (1). The percentage uncertainty in g equals the
percentage uncertainty in the gradient, $(\Delta\text{gradient}/\text{gradient}) \times 100$, since g is inversely
proportional to the gradient (1).
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